# How do Planets Really Move – Kepler’s Laws

Although gravity is the weakest of the known forces, it dominates the Universe on large scales. The sub-atomic “strong” and “weak” forces work only over very small distances, the size of or smaller than an atomic nucleus. We didn’t even know these two forces existed until the 20th century. The electromagnetic force is quite strong, far stronger than gravity. But mass, the source of gravity, comes in only one form while electric charge, the source of electromagnetism, comes in two, positive and negative. Electromagnetism, despite its strength, is not important on the large scale because most of the matter in the universe contains exactly equal amounts of positive and negative charge, and they cancel out each other’s effects. Understanding gravity is essential to understanding the Universe.

One of the most important events in developing an understanding of gravity was Kepler’s discovery that the orbits of the planets obey three laws, one about the shape of the orbit, one about how fast a planet moves around that orbit, and one about how the size of the orbit and the time to go around it once are related to the masses. After Isaac Newton’s discovery of the law of universal gravitation, we now know that Kepler’s laws apply to any two bodies in an orbit, including stars.

**How Kepler Cracked the Problem of Orbits**

Consider the problem Kepler had to solve: he is sitting on Earth, a planet with an unknown orbit. He is looking at another planet, say Mars, also with an unknown orbit. Knowing nothing other than the positions of Mars in the sky of Earth, work out both orbits. Kepler knew nothing of gravity or the laws of motion. Those came later. But he did have an enormous catalog of precise measurements of planetary positions covering decades that was accumulated by Tycho Brahe, who employed Kepler.

Kepler’s method was genius in its simplicity. Instead of looking at the whole great pile of B rahe’s observations all at once, pick out only those that were made on the same day in different years. The Earth is always at the same place in its orbit on that day so effectively you have frozen the Earth in place. Any changes in the position of Mars on those days must therefore be due to its motion alone and you can work out its orbit. Then, having Mars’ orbit in hand, do the same trick backwards. Select only observations made one Martian year apart. Now Mars is frozen and any differences in its position must be due to the orbital motion of the Earth. For an encore, do this for all the other planets and do so while you are on the road running away from the Thirty Years’ War.

**Part 1: What shape are orbits?**

Start the NAAP labs app that you should have installed and choose 5. Planetary Orbits. Let’s play around with it a bit.

The first two items on the menu are explanations of Kepler’s Laws and Newton’s extension of them. These are worth reading, but not essential. Kepler’s first law says orbits are ellipses with the Sun at one focus. Ellipses are an oval shape, but a special kind of oval. You can think of it as a sort of squashed circle with two centers, called the foci.

Select the Planetary Orbit Simulator

* Open the Kepler’s 1st Law tab if it is not already open (it opens by default)
* Enable all five check boxes (empty focus, center, semi-major axis, semi-minor axis, and radial lines)
* The white dot is the “simulated planet.” Click on it to drag it around the orbit. At the bottom of the simulator, you will see an equation relating r1 and r2 to 2a. “a” is the letter commonly used to refer to the semi-major axis, which is half the side to side distance the long way.
* Change the size of the orbit with the semi-major axis slider. Note how the background grid indicates change in scale while the displayed orbit size remains the same (for this simulation, the size of the semi-major axis is limited to 50 AU).
* Change the eccentricity and note how it affects the shape of the orbit (eccentricities are limited to be 0.7 or less, and the semi-major axis is always aligned horizontally, unlike real planets).
* Animate the simulated planet. You may need to increase the animation rate for very large orbits or decrease it for small ones.
* Compare the orbits of the planets of our solar system using the planet presets.

1. For what eccentricity is the secondary focus (which is usually empty) located at the sun? What is the shape of this orbit?

B. Create an orbit with a = 20 AU and e = 0. “e” is called the eccentricity and measures how squashed the ellipse is. Drag the planet first to the far left of the ellipse and then to the far right. What are the values of r1 and r2 at these locations? What kind of shape is this?

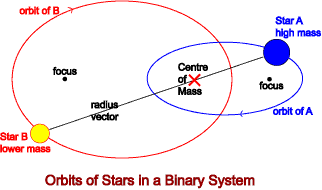
C. Create an orbit with a = 20 AU and e = 0.5. Drag the planet first to the far left of the ellipse and then to the far right. What are the values of r1 and r2 at these locations?

D. For the ellipse with a = 20 AU and e = 0.5, can you find a point in the orbit where r1 and r2 are equal? Sketch the ellipse, the location of this point, and r1 and r2 in the space below.

E. What is the value of the sum of r1 and r2 and how does it relate to the ellipse properties? Is this true for all ellipses? (This provides an easy way to draw ellipses. After you have answered, you can look at the animation “how to draw an ellipse.gif” on D2L.)

The Sun also has an elliptical orbit. It is just really really tiny (a really really small semi-major axis a). This is because the Sun is huge compared to any of the planets.

About half of the stars in the universe are part of multiple star systems, and most of those are binary star systems. Their orbits look like this:



F. Does the location of the center of mass (the “balance point”) correspond to any special point on each of the ellipses?

You can find a simulator here that allows you to play with the relative masses and eccentricity:

<http://www.astro.ucla.edu/undergrad/astro3/orbits.html>

At this site, you can see some orbits for 3 and 4 star systems as well. There is a lot more variety in what they can do compared to binaries.

<http://www.atlasoftheuniverse.com/orbits.html>

G. Explore the orbital parameters (semi-major axis and eccentricity) of several nearby binary star systems, available on the website [https://web.archive.org/web/20210526003406/http://www.solstation.com/orbits.htm#sthash.rFohrMEy.rFohrMEy.dpbs](https://web.archive.org/web/20210526003406/http:/www.solstation.com/orbits.htm#sthash.rFohrMEy.rFohrMEy.dpbs) Good examples are Alpha Centauri AB, Sirius AB, 61 Cygni AB, and 36 Ophiuchi AB, but anything labelled AB is a binary. Each letter designates one of the two stars. Try entering a few in the simulator in question F above. What values of eccentricity are typical for binary star systems? Are circular orbits common or rare?

**Discuss your results with your instructor at this point**

**Part 2: Kepler's Second Law**

* Use the “clear optional features” button to remove the 1st Law features.
* Click on the Kepler's 2nd Law tab
* Press the “start sweeping” button. Adjust the semi-major axis and animation rate so that the planet moves at a reasonable speed.
* Adjust the size of the sweep using the “adjust size” slider.
* Click and drag the sweep segment around. Note how the shape of the sweep segment changes, but the area does not (you can find the area of the sweep at the bottom).
* Add more sweeps. Erase all sweeps with the “erase sweeps” button.
* The “sweep continuously” check box will cause sweeps to be created continuously when sweeping. Test this option.

A. In the NAAP simulator, erase all sweeps and create an ellipse with a = 1 AU and e = 0. Set the fractional sweep size to one-twelfth of the period and create a sweep segment. Drag the sweep segment around. Does the size or shape of the sweep segment change as it moves around the orbit? Why or why not?

B. Select the orbit preset for the planet Mercury. Where in its orbit does Mercury move the fastest? Where in its orbit does Mercury spend the most time? Explain your answers.

C. Set the semi-major axis back to 1 AU and change the eccentricity to e = 0.5. Drag the sweep segment around and note that its size and shape change. Where is the sweep segment the “skinniest?” Where is it the “fattest?” Where is the planet when it is sweeping out each of these segments? What can you say about the area for these sweeps? Explain your answers.

D. What eccentricity in the simulator gives the greatest variation of sweep segment shape?

Halley’s comet has a semi-major axis of about 18.5 AU, a period of 76 years, and an eccentricity of about 0.97 (Halley’s orbit cannot be shown in this simulator since it only goes up to e = 0.7).

E. The orbit of Halley’s Comet, the Earth’s Orbit, and the Sun are shown in the diagram below (but not even close to scale). Based upon what you now know about Kepler’s 2nd Law, explain why we can only see the comet for about 6 months out of each 76 year orbit.

**Discuss your results with your instructor at this point**

**Part 3: Kepler’s Third Law**

Explore the relationship between a planet’s orbital period and its semi-major axis. The relationship is known as Kepler’s Third Law. The graph plots the orbital period (p) as a function of semi-major axis. Click the “up/down” buttons to explore what mathematical relationship that will fit the orbits of planets in the Solar System.

* Use the “clear optional features” button to remove the 2nd Law features.
* Open the Kepler's 3rd Law tab.

A. Use the simulator to complete the table below. For the last two, it will take a few tries.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Object | P (years) | a (AU) | e | P2 | a3 |
| Earth |  | 1.00 |  |  |  |
| Mars |  | 1.52 |  |  |  |
| Ceres |  | 2.77 | 0.08 |  |  |
| Chiron | 50.7 |  | 0.38 |  |  |

B. As the size of a planet’s orbit increases, what happens to its period? Do they seem to be directly related (that is, is p/a always about the same number)?

C. Start with the Earth’s orbit and change the eccentricity to 0.6. Does changing the eccentricity change the period of the planet?

So now let’s weigh the sun. Once the Earth–Sun distance is known, the period of the Earth’s orbit gives the mass of the Sun directly from Newton’s form of Kepler’s third law[[1]](#footnote-1).

The original form of Kepler’s third law gives masses in solar masses; knowing that the mass of the Sun is 1 solar mass is not enlightening! It is exactly one of itself. I guess it wouldn’t lie about a thing like that. However, Kepler’s third law can be directly derived from Newton’s law of gravity (see chapter 2 for details), and this version will use period in seconds, semi-major axis in meters, and mass in kilograms. The disadvantage is the inconvenience of the units and the need to know G, the universal gravitational constant. In these units, G = 6.67 × 10−11.

D. The table in Appendix F gives the masses of the planets in units of Earth masses. Appendix E will give you the mass of the Sun. Looking at these masses, can you justify ignoring *Mplanet* in this equation?

E. Using P = 1 year = 31,536,000 s and a = 1 AU = 1.496 × 1011 m what do you get for the mass of the Sun in kilograms? Explain your work.

That is a very large number, and many stars are much more massive. Therefore, expressing the masses of stars in multiples of the Sun’s mass instead of kilograms makes the numbers easier to comprehend. This leads to the form of Kepler’s third law that you see at the bottom of the NAAP simulation.

*MP2 = a3* units: a in astronomical units (AU; one AU is the semimajor axis of Earth’s orbit)

P in years

M in solar masses

For a binary star system, Kepler’s Third Law can be written:

or, using solar units

But now you can’t just ignore one of the masses because they are both large. So now there are two unknown masses, and you can’t find either of them with just one equation. We need another constraint.

That problem is considerably more complicated, but it boils down to being able to measure either the velocities of the stars, or their respective distances from their center of mass. We might come back to that later but this is already long enough.

**Discuss your results with your instructor at this point**

**Part 4: Weighing the galaxy**

**Orbit of the Sun around the Galaxy**

The Sun follows a (mostly) circular orbit around the center of the Milky Way with an orbital speed of 220 km s-1. The distance to the Galactic Center is a mere 26,000 light years (2.4 x 1017 km or 1.7 x 109 AU).

A. How many years does it take the Sun to complete one full orbit around the Milky Way? Show your work.

B. We know from radiooactive dating and stellar evolution models that the Sun formed 4.6 billion years ago. How many orbits has the Sun completed around the Galaxy since the Sun formed? Explain your answer.

C. Estimate the mass of the Milky Way from the Sun’s orbit using Kepler’s law. (This is just the amount of mass interior to the Sun’s orbit. The mass further out than the Sun does not affect its orbit, and there is a really interesting explanation of why.) Explain your answer.

1. Kepler knew that *P2* was proportional to *a3* ; that is, *P2* = K*a3* where K is some constant number but he didn’t know *why* it was that number or what that number meant. Newton’s later work showed that it depended on the masses and a couple of universal constants. [↑](#footnote-ref-1)